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Simple scheme for implementing the Deutsch–Jozsa algorithm in a thermal cavity

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Abstract

We present a simple scheme to implement the Deutsch–Jozsa algorithm based on two-atom interaction in a thermal cavity. The photon-number-dependent parts in the evolution operator are cancelled with the strong resonant classical field added. As a result, our scheme is immune to the thermal field, and does not require the cavity to remain in the vacuum state throughout the procedure. Besides, large detuning between the atoms and the cavity is not necessary either, leading to potential speed up of quantum operation. Finally, we show by numerical simulation that the proposed scheme is equal to demonstrate the Deutsch–Jozsa algorithm with high fidelity.

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(Some figures in this article are in colour only in the electronic version)

Ever since Feynman first pointed out the concept of a quantum computer in 1982 [1], quantum computation has undergone rapid progress. The new type of computer can solve some problems much faster than its classical counterpart. For example, the well-known Deutsch–Jozsa problem, which is to identify whether a binary-valued function $f(x)$ of N bits variables is *constant* for all values of x, or *balanced* (equal to 1 for exactly half of all the possible x, and 0 for the other half), can be solved by using a single query of $f(x)$ on a quantum computer [2, 3], whereas a classical computer needs up to $(2^{N-1}+1)$ queries [4]. Up til now, the Deutsch–Jozsa algorithm has been widely studied [5–9], with its efficiency experimentally tested [10, 11].

The Deutsch–Jozsa algorithm can be briefly described as follows. Assume for the simplest case that $f(x)$ has only a one-bit input ($x = 0$ or 1). The algorithm can be performed in a

system consisting of one query qubit 1 and an auxiliary working qubit 2, which are initially prepared in the superposed state:

$$|\psi\rangle_1 = \tfrac{1}{2}(|0\rangle_1 + |1\rangle_1)(|0\rangle_2 - |1\rangle_2). \tag{1}$$

Then a unitary operator U_{fn} is used to calculate $f(x)$, which acts as $U_{fn}|x\rangle_1|y\rangle_2 = |x\rangle_1|y \oplus f(x)\rangle_2$. Here $x, y \in \{0, 1\}$ and $\oplus$ indicates addition modulo 2. The implementation of U_{fn} on $|\psi_1\rangle$ yields

$$|\psi\rangle_2 = \tfrac{1}{2}[(-1)^{f(0)}|0\rangle_A + (-1)^{f(1)}|1\rangle_A](|0\rangle_B - |1\rangle_B). \tag{2}$$

There are actually four possible U_{fn}: U_{f1} corresponds to $f(0) = f(1) = 0$; U_{f2} corresponds to $f(0) = f(1) = 1$; U_{f3} corresponds to $f(0) = 0$ and $f(1) = 1$; U_{f4} corresponds to $f(0) = 1$ and $f(1) = 0$; after performing a Hadamard transformation on the query qubit, the state of qubit 1 will be in $|0\rangle$ for U_{f1} and U_{f2}, while for U_{f3} and U_{f4} it becomes $|1\rangle$. Thus a measurement on the query qubit will tell us whether $f(x)$ is constant or balanced.

Recently, Zheng proposed a scheme [12] for implementing the Deutsch–Jozsa algorithm in cavity QED, in which two atoms, one as a preparing qubit and one as the query bit, sequentially interact with the cavity mode, which serves as the auxiliary working qubit. The main drawback of this scheme, as well as several other implementations of quantum algorithms using cavity QED [13, 14], is that they are sensitive to cavity decay or the thermal field, which makes the practical experiment difficult to be scalable. Although the decoherence time of the cavity can be prolonged by keeping the cavity excited merely in a virtual way [15], one has to make sure that the cavity remains always in the vacuum state throughout the procedure, otherwise it is still sensitive to the thermal field [16].

We note that by resorting to a strong classical field, such drawbacks can be overcome [17] in that the photon-number-dependent parts in the evolution operator are cancelled with the resonant classical field added, thus rending immunity to the thermal field. Based on the state evolution presented in [17], we propose an improved scheme for implementing the Deutsch–Jozsa algorithm which outruns the previous scheme in [12] in three important aspects: (1) both the preparing atom and the auxiliary atom level used in [12] are not necessary; instead we only use two two-level atoms, which might simplify the experimental procedures effectively if scalability is concerned; (2) cavity is not required to remain in the vacuum state all the time, and insensitivity to the thermal field is still insured; (3) no large detuning between atoms and the cavity is necessary, potentially giving rise to a better performance of the speed and fidelity of the whole procedure.

To describe our scheme, let us first consider two identical two-level atoms simultaneously interacting with a single-mode cavity field and driven by a classical field. The Hamiltonian (assuming $\hbar = 1$) in the rotating-wave approximation reads [17–19]

$$H = \frac{1}{2}\sum_{j=1}^{2}\omega_0\sigma_{z,j} + \omega_a a^\dagger a + \sum_{j=1}^{2}\left[g\left(a^\dagger\sigma_j^- + a\sigma_j^+\right) + \Omega\left(\sigma_j^+ \mathrm{e}^{-\mathrm{i}\omega t} + \sigma_j^- \mathrm{e}^{\mathrm{i}\omega t}\right)\right], \tag{3}$$

where $\sigma_{z,j} = |e\rangle_{jj}\langle e| - |g\rangle_{jj}\langle g|$, $\sigma_j^+ = |e\rangle_{jj}\langle g|$, $\sigma_j^- = |g\rangle_{jj}\langle e|$, with $|e\rangle_j(|g\rangle_j)$ being the excited (ground) state of the jth atom. ω_0, ω_a and ω are the frequencies for atomic transition, cavity mode and classical field, respectively. $a^\dagger$ and a are the creation and annihilation operators for the cavity mode. g is the atom–cavity coupling strength and Ω is the Rabi frequency of the classical field. Assume that $\omega_0 = \omega$. Then we can obtain the following interaction Hamiltonian in the interaction picture:

$$H_I = \sum_{j=1}^{2}\left[\Omega\left(\sigma_j^+ + \sigma_j^-\right) + g\left(\mathrm{e}^{-\mathrm{i}\delta t}a^\dagger\sigma_j^- + \mathrm{e}^{\mathrm{i}\delta t}a\sigma_j^+\right)\right], \tag{4}$$

where $\delta = \omega_0 - \omega_a$. For the new atomic basis $|\pm\rangle_j = (|g\rangle_j \pm |e\rangle_j)/\sqrt{2}$, we can rewrite $H_I = H_e + H_0$ with

$$H_0 = \sum_{j=1}^{2} 2\Omega S_{z,j}, \tag{5}$$

$$H_e = \sum_{j=1}^{2} g\left[\mathrm{e}^{-\mathrm{i}\delta t} a^{\dagger}\left(S_{z,j} + \frac{1}{2}S_j^{-} - \frac{1}{2}S_j^{+}\right) + \mathrm{H.c.}\right]. \tag{6}$$

Here $S_{z,j} = (|+\rangle_{jj}\langle +| - |-\rangle_{jj}\langle -|)/2$, $S_j^{+} = |+\rangle_{jj}\langle -|$ and $S_j^{-} = |-\rangle_{jj}\langle +|$. Assuming that $\Omega \gg \delta, g$, we can neglect the fast oscillating terms. Then the effective Hamiltonian H_e reduces to

$$H_e = g(\mathrm{e}^{-\mathrm{i}\delta t} a^{\dagger} + \mathrm{e}^{\mathrm{i}\delta t} a)\sigma_x, \tag{7}$$

where $\sigma_x = \frac{1}{2}\sum_{j=1}^{2}\left(\sigma_j^{+} + \sigma_j^{-}\right)$. The evolution operator for Hamiltonian (equation (7)) can be written as

$$U_e(t) = \mathrm{e}^{-\mathrm{i}A(t)\sigma_x^2}\,\mathrm{e}^{-\mathrm{i}B(t)\sigma_x a}\,\mathrm{e}^{-\mathrm{i}C(t)\sigma_x a^{\dagger}}, \tag{8}$$

which was first proposed for a trapped-ion system [20]. By solving the Schrödinger equation $\mathrm{i}\frac{\mathrm{d}U_e(t)}{\mathrm{d}t} = H_I U_e(t)$, we can obtain $B(t) = g(\mathrm{e}^{\mathrm{i}\delta t} - 1)/\mathrm{i}\delta$, $C(t) = -g(\mathrm{e}^{-\mathrm{i}\delta t} - 1)/\mathrm{i}\delta$ and $A(t) = g^2[t + (\mathrm{e}^{-\mathrm{i}\delta t} - 1)/\mathrm{i}\delta]/\delta$. Setting $\delta t = 2\pi$, we have $B(t) = C(t) = 0$. Then we can get the evolution operator of the system

$$U_I(t) = \mathrm{e}^{-\mathrm{i}H_0 t} U_e(t) = \mathrm{e}^{-\mathrm{i}2\Omega\sigma_x t - \mathrm{i}2\lambda\sigma_x^2 t}, \tag{9}$$

with $\lambda = g^2/2\delta$. We note that the evolution operator is independent of the cavity field state, allowing it to be in a thermal state. Unlike [15], our scheme does not require $\delta \gg g$. The atoms interact with the cavity mode for a time t, leading to

$$\begin{aligned}|e\rangle_1|g\rangle_2 \to\ & \mathrm{e}^{-\mathrm{i}\lambda t}\{\cos(\lambda t)[\cos(\Omega t)|e\rangle_1 - \mathrm{i}\sin(\Omega t)|g\rangle_1] \\ & \times [\cos(\Omega t)|g\rangle_2 - \mathrm{i}\sin(\Omega t)|e\rangle_2] - \mathrm{i}\sin(\lambda t) \\ & \times [\cos(\Omega t)|g\rangle_1 - \mathrm{i}\sin(\Omega t)|e\rangle_1][\cos(\Omega t)|e\rangle_2 \\ & - \mathrm{i}\sin(\Omega t)|g\rangle_2]\}\end{aligned} \tag{10}$$

and

$$\begin{aligned}|g\rangle_1|g\rangle_2 \to\ & \mathrm{e}^{-\mathrm{i}\lambda t}\{\cos(\lambda t)[\cos(\Omega t)|g\rangle_1 - \mathrm{i}\sin(\Omega t)|e\rangle_1] \\ & \times [\cos(\Omega t)|g\rangle_2 - \mathrm{i}\sin(\Omega t)|e\rangle_2] - \mathrm{i}\sin(\lambda t) \\ & \times [\cos(\Omega t)|e\rangle_1 - \mathrm{i}\sin(\Omega t)|g\rangle_1][\cos(\Omega t)|e\rangle_2 \\ & - \mathrm{i}\sin(\Omega t)|g\rangle_2]\}.\end{aligned} \tag{11}$$

We choose the interaction time and Rabi frequency appropriately so that $\Omega t = (2m+1)\pi$ (m is integer) and $\lambda t = \pi/4$, then we obtain the Einstein–Podolsky–Rosen state (EPR state),

$$\tfrac{1}{\sqrt{2}}(|e\rangle_1|g\rangle_2 - \mathrm{i}|g\rangle_1|e\rangle_2), \tag{12}$$

and

$$\tfrac{1}{\sqrt{2}}(|g\rangle_1|g\rangle_2 - \mathrm{i}|e\rangle_1|e\rangle_2), \tag{13}$$

where we have discarded the common phase factor. These calculations are useful later in numerical analysis of fidelity.

Now we would like to show that with the help of $U_I(t)$ as shown in equation (9), all the relevant unitary operators U_{fn} in the Deutsch–Jozsa algorithm can be easily achieved. But before that, we first need to show how to use the above idea to realize the quantum CNOT gate. For the present problem, we let the first atom serve as the query qubit and the second atom as the auxiliary working qubit, i.e. $|g\rangle_i$ and $|e\rangle_i$, respectively, represent $|0\rangle_i$ and $|1\rangle_i$ $(i = 1, 2)$ in equation (1).

According to equation (9), it can be easily shown that

$$\begin{cases} U_I(t)|+\rangle_1|+\rangle_2 = \mathrm{e}^{-\mathrm{i}2(\Omega+\lambda)t}|+\rangle_1|+\rangle_2 \\ U_I(t)|+\rangle_1|-\rangle_2 = |+\rangle_1|-\rangle_2 \\ U_I(t)|-\rangle_1|+\rangle_2 = |-\rangle_1|+\rangle_2 \\ U_I(t)|-\rangle_1|-\rangle_2 = \mathrm{e}^{-\mathrm{i}2(\Omega-\lambda)t}|-\rangle_1|-\rangle_2. \end{cases} \tag{14}$$

By setting $\delta = \sqrt{2}g$ and $gt = 2\pi$, we can make the interacting time t and Rabi frequency Ω satisfy $\lambda t = \pi/2$ and $\Omega t = \left(2k + \frac{1}{2}\right)\pi$ (k is an integer). Then we have

$$\begin{cases} U_I(t)|+\rangle_1|+\rangle_2 = -|+\rangle_1|+\rangle_2 \\ U_I(t)|+\rangle_1|-\rangle_2 = |+\rangle_1|-\rangle_2 \\ U_I(t)|-\rangle_1|+\rangle_2 = |-\rangle_1|+\rangle_2 \\ U_I(t)|-\rangle_1|-\rangle_2 = |-\rangle_1|-\rangle_2. \end{cases} \tag{15}$$

As a result, we obtain a controlled-phase gate, which can be transformed into a CNOT gate through a few single-qubit unitary operations:

$$\begin{aligned} & \overset{X_1}{\rightarrow} \quad \overset{H_1}{\rightarrow} \quad \overset{U_I(t)}{\rightarrow} \quad \overset{H_1}{\rightarrow} \quad \overset{X_1}{\rightarrow} \quad \overset{Z_1}{\rightarrow} \\ & |g\rangle_1|g\rangle_2 \rightarrow |e\rangle_1|g\rangle_2 \rightarrow |-\rangle_1|g\rangle_2 \rightarrow |-\rangle_1|g\rangle_2 \rightarrow |e\rangle_1|g\rangle_2 \rightarrow |g\rangle_1|g\rangle_2 \rightarrow |g\rangle_1|g\rangle_2 \\ & |g\rangle_1|e\rangle_2 \rightarrow |e\rangle_1|e\rangle_2 \rightarrow |-\rangle_1|e\rangle_2 \rightarrow |-\rangle_1|e\rangle_2 \rightarrow |e\rangle_1|e\rangle_2 \rightarrow |g\rangle_1|e\rangle_2 \rightarrow |g\rangle_1|e\rangle_2 \\ & |e\rangle_1|g\rangle_2 \rightarrow |g\rangle_1|g\rangle_2 \rightarrow |+\rangle_1|g\rangle_2 \rightarrow -|+\rangle_1|e\rangle_2 \rightarrow -|g\rangle_1|e\rangle_2 \rightarrow -|e\rangle_1|e\rangle_2 \rightarrow |e\rangle_1|e\rangle_2 \\ & |e\rangle_1|e\rangle_2 \rightarrow |g\rangle_1|e\rangle_2 \rightarrow |+\rangle_1|e\rangle_2 \rightarrow -|+\rangle_1|g\rangle_2 \rightarrow -|g\rangle_1|g\rangle_2 \rightarrow -|e\rangle_1|g\rangle_2 \rightarrow |e\rangle_1|g\rangle_2. \end{aligned} \tag{16}$$

Here H_1, X_1 and Z_1 are Hadamard, σ_x and σ_z operations on the first atom with computational basis being $|g\rangle_1$ and $|e\rangle_1$, which can be easily realized by choosing the appropriate amplitudes and phases of classical fields, respectively.

U_{f1} operation. This operation on the atomic qubits does not require any interaction with the cavity mode. In this case the atoms can be tuned far off resonant with the cavity mode and thus the atom–cavity evolution is freezing. Thus the system remains in the state of $|\psi\rangle_1$.

U_{f2} operation. We first apply the aforementioned CNOT gate, and then perform the single-qubit transformation $|g\rangle_1 \rightarrow |e\rangle_1$ and $|e\rangle_1 \rightarrow -|g\rangle_1$ on the atom A by using a π-Ramsey pulse. Then we repeat the controlled-NOT operation and perform the transformation $|g\rangle_1 \rightarrow -|e\rangle_1$ and $|e\rangle_1 \rightarrow |g\rangle_1$ by using a π-Ramsey pulse with a phase difference π relative to the first Ramsey pulse. Therefore we obtain

$$\begin{aligned} |\psi\rangle_2 &= \tfrac{1}{2}(|0\rangle_1 + |1\rangle_1)(|0 \oplus 1\rangle_2 - |1 \oplus 1\rangle_2) \\ &= \tfrac{1}{2}(-|0\rangle_1 - |1\rangle_1)(|0\rangle_2 - |1\rangle_2). \end{aligned} \tag{17}$$

U_{f3} operation. By performing the CNOT gate operation of equation (16), the output state of two atoms is given by

$$\begin{aligned} |\psi\rangle_2 &= \tfrac{1}{2}[|0\rangle_1(|0\rangle_2 - |1\rangle_2) + |1\rangle_1(|0 \oplus 1\rangle_2 - |1 \oplus 1\rangle_2)] \\ &= \tfrac{1}{2}(|0\rangle_1 - |1\rangle_1)(|0\rangle_2 - |1\rangle_2). \end{aligned} \tag{18}$$

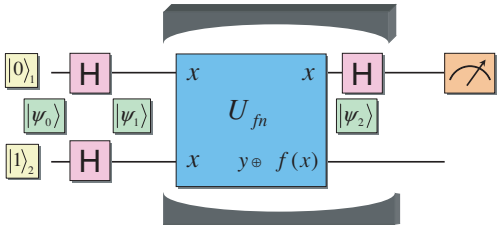

Figure 1. Experimental apparatus of the whole scheme, where atoms A and B cross the cavity with same velocity but at different positions, allowing for single-qubit operation of each one in the process.

U_{f4} operation. Firstly, we perform the single-qubit transformation $|g\rangle_1 \rightarrow |e\rangle_1$ and $|e\rangle_1 \rightarrow -|g\rangle_1$ on the atom A; secondly, the CNOT operation of equation (16) is applied; finally, we perform the single-qubit transformation $|g\rangle_1 \rightarrow -|e\rangle_1$ and $|e\rangle_1 \rightarrow |g\rangle_1$. This leads to

$$\begin{aligned} |\psi\rangle_2 &= \tfrac{1}{2}[|1\rangle_1(|0\rangle_2 - |1\rangle_2) + |0\rangle_1(|0 \oplus 1\rangle_2 - |1 \oplus 1\rangle_2)] \\ &= \tfrac{1}{2}(-|0\rangle_1 + |1\rangle_1)(|0\rangle_2 - |1\rangle_2). \end{aligned} \tag{19}$$

The whole scheme of the implementation of the Deutsch–Jozsa algorithm is displayed in figure 1. Two atoms 1 and 2, first simultaneously prepared in a box into a high lying circular Rydberg state denoted by $|g\rangle_1|e\rangle_2$, are in the initial average state after a Hadamard operation. Then they undergo the operations in figure 1 from left to right. In order to realize the different operations U_{fn}, we have to employ an inhomogeneous field to distinguish the two atoms by the same trick as in [13]. Finally, atoms 1 and 2 are separately read out by the state-selective field-ionization detectors.

We briefly discuss the experimental feasibility of our proposal. Although the evolution operator is independent of the thermal photons of a cavity field as decided by the condition $\delta t = 2\pi$, the two-atom system is entangled with the cavity during the atom–cavity interaction. We have to neglect the cavity decay during this interaction time. We assume that the atom–cavity coupling constant is $g = 2\pi \times 25$ kHz [21, 22], $\delta = \sqrt{2}g$. Direct calculation shows that the interaction time is at the order of 10^{-5} s. Note that the photon decay time is $T_c \simeq 10^{-3}$ s, thus much longer than the interaction time. After the interaction, the atoms are disentangled with the cavity, that is, the operation will not be affected by the cavity decay during the interaction time. Besides, the radiative time for the Rydberg atoms is $T_r = 3 \times 10^{-2}$ s and the implementation time needed to complete the whole procedure in the cavity is much shorter than T_r as the time for single-qubit transformation is negligible. Thus the proposed scheme is realizable with the present cavity QED techniques. The most probable difficulty is to send two atoms simultaneously through the cavity, but other works have shown that even though there exist time difference, the negative influence is almost negligible. [15, 18].

In obtaining equation (7), we have discarded the fast oscillating terms, which induce Stark shifts on the states $|+\rangle_j$ and $|-\rangle_j$. Here we numerically simulate the dependence of fidelity considering the error introduced by the Stark shift for generations of EPR state as in equation (12) with different values of detuning, as shown in figure 2(*a*). (Note that we have set $\Omega = 20\delta$). The result from the plot shows that even for $\delta = \sqrt{2}g$, the fidelity is still larger than 97%, from which we know that large detuning is not required in our scheme. Besides, if

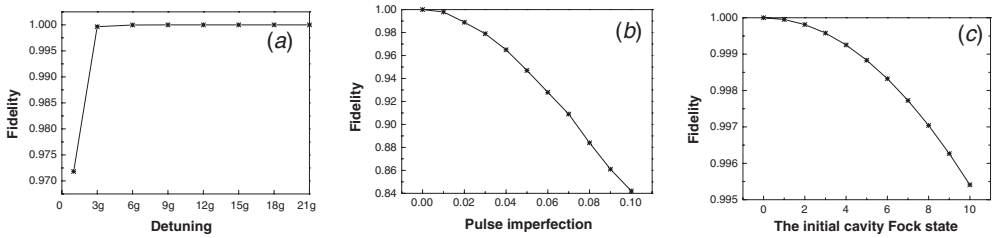

Figure 2. Numerical results for the fidelity of our scheme: (*a*) error introduced by the Stark Shifts ($\Omega = 20\delta$); (*b*), (*c*) show the dependence of the fidelity on pulse imperfections and initial cavity Fock state ($g = 2\pi \times 25$ kHz, $\delta = 20 \times g$, $\Omega = 400 \times g$).

we consider the fluctuation of the Rabi frequency $\Delta\Omega = 0.01\Omega$, direct calculation shows that the fidelity for the generation of EPR state decreases by only 0.02.

To check the feasibility of our scheme more strictly, we show in figure 2(*b*) the estimated achievable fidelity of the produced EPR state, where we consider the existence of fluctuations in the atom–cavity interaction that leads to imperfections of the quantum Rabi pulses [23]. The fidelity is plotted for various strengths of imperfections in the Rabi pulses, where we assume for simplicity that the initial cavity state is in $|5\rangle$ and each pulse suffers the same imperfection. The result from the plot tells that even for 10% pulse error, the fidelity is still larger than 80%. (Note that in real experiments this kind of imperfection can be controlled around 3%.)

Furthermore, we show in figure 2(*c*) that if we assume that the cavity is initially in a Fock state $|n\rangle$, the success probability for producing the EPR state slightly decreases with the increase of photon number. Even for $n = 10$, the fidelity is still larger than 99.5%, which means that the whole process is almost independent of the cavity field state.

In principle, our scheme may offer a viable way to realize a scalable quantum algorithm. Based on the effective interaction between two atoms with single-mode cavity, our scheme can be also extended to a multi-bit Deutsch–Jozsa algorithm, since the multi-bit transformation U'_{fn} can be constructed by single-qubit quantum gates and CNOT gates [24]. However, it is still somewhat difficult for our scheme to be extended to many qubits based on the current technology [22]. Furthermore, it should be pointed out that the single-atom sources are required in our scheme.

To sum up, we have proposed a simple scheme for implementing the Deutsch–Jozsa algorithm in cavity QED based on the effective interaction of two two-level atoms with a single-mode cavity with the assistance of a strong classical driving field. Compared with the scheme in [12], our scheme is immune to the thermal field and does not require the cavity to remain in a vacuum state. In addition, the scheme may work in a fast way since large detuning is not required. Besides, our scheme does not require the auxiliary atom level for the implementation of the quantum CNOT operation and all the operations except the single-qubit transformation are imposed on both atoms simultaneously, making our scheme easier to be carried out practically. Based on these features, we present finally the numerical analysis of the fidelity of our scheme with respect to the practical experiment under the influence of detuning, pulse imperfection and the initial cavity Fock state.

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